Figures 3-5 show the results of calculations of the amplitudes of perturbations of the boundaries of a cylindrical layer. In all of the figures, curve 1 corresponds to $R^1_{n\lambda}$, while curve 2 corresponds to $R^2_{n\lambda}$. Line 3 describes the behavior of the radius of the internal cavity r_1, $\mu_* = (1 + kt_*)^2$, $\varepsilon = 8$, $S_1 = S_2 = 10$. Figure 3 shows the behavior of two-dimensional perturbations (23), (27) with $\lambda = 1, 2, 3$, while Fig. 4 shows the characteristic curves for radial perturbations (23), (28). Figure 5 illustrates the behavior of perturbations at the free surfaces of the layer with $q = 1$, $\lambda = 0, 1, 2, 3$. The numerical curves that were constructed confirm the asymptotic results of the previous section — the internal surface is unstable during collapse, while the perturbations on the external surface die out.

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EFFECT OF THE CHOICE OF CREEP INSTABILITY CRITERION ON THE SOLUTION OF THE PROBLEM OF OPTIMIZING ROD-SHAPED STRUCTURES

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There are several approaches to evaluating the stability of a structure under creep conditions [1]. Uncertainty in the selection of a criterion of instability is an obstacle to the exact formulation of the problem of optimizing rheological systems. None of the existing solutions [2, 3] combine the results of solution of the problem for different approaches. Such a combination is lacking despite the fact that these approaches differ significantly in regard to their value for predicting the critical time.

The goal of the present study is to evaluate the effect of the choice of instability criterion on the solution of the optimization problem. We will examine so-called conditional criteria [4]. We present the equations of the problem of the maximum of the critical time for an arbitrary rod-shaped structure, and we use a specific example to determine the condition of the minimum of volume for a fixed critical time. It is shown that the choice of criterion has no effect on the optimum form of the system in the first case and that the effect is negligible in the second case.

We will assume that the material of the rod obeys the creep law [5]

$$\dot{p}p^{\alpha} = f(\sigma) \tag{1}$$

($p = \varepsilon - \sigma/E$ is the creep strain; α is the strain-hardening parameter). Analyzing variants of conditional instability criteria for creep, we note that for most of them the critical strain for a compressed rod can be represented in the form

$$p = \varphi(\sigma_0 - \sigma)/E, \tag{2}$$

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TABLE 1

Criterion	φ			β
	$f(\sigma)$	$f=A\sigma^n$	$\alpha=2/3,\ n=5$	
[8]	$\frac{\alpha f(\sigma)}{\sigma f'(\sigma)}$	$\frac{\alpha}{n}$	0,133	35°07′
[4]	$\frac{2\alpha f(\sigma)}{\sigma f'(\sigma)}$	$\frac{2\alpha}{n}$	0,266	35°04′
[10]	$\frac{(1+\alpha)f(\sigma)}{\sigma f'(\sigma)}$	$\frac{1+\alpha}{n}$	0,333	35°03′
[6]	$\frac{(1+\alpha)f(\sigma_0)}{\sigma_0 f'(\sigma_0)}$	$\frac{1+\alpha}{n}$	0,333	35°03′
[9]	$\frac{3\alpha f(\sigma)}{\sigma f'(\sigma)}$	$\frac{3\alpha}{n}$	0,400	35°02′
[12]	$\frac{(1+2\alpha)f(\sigma)}{\sigma f'(\sigma)}$	$\frac{1+2\alpha}{n}$	0,466	35°01′
[11]	1	1	1,000	34°54′

where φ depends on the chosen criterion (see Table 1); $\sigma_0 = (\pi/\ell)^2EI/F$ is the Eulerian critical stress of the electric rod; I and F are the moment of inertia and the cross-sectional area. The rods in the structure are assumed to be hinged. The criterion in [6] is represented approximately in the form (2) at stresses close to σ_0.*

With the power law $f = A\sigma^n$, the coefficient φ is a constant which depends only on the properties of the material (see Table 1). Ignoring the change in the geometry of the system over time, we will assume that for constant loads the stresses in the rods are also constant. Integrating (1) with power relation $f(\sigma)$, we obtain

$$p = (At(\alpha+1))^{\gamma/n},\ \gamma = n/(1+\alpha). \tag{3}$$

We introduce the dimensionless time parameter

$$\tau = (At(\alpha+1)E^n)^{\gamma/n}/\varphi. \tag{4}$$

With allowance for the latter, Eqs. (2) and (3) give

$$\sigma - \sigma_0 + \tau\sigma^\gamma E^{1-\gamma} = 0. \tag{5}$$

We will express the stress in the rod with the number j ($j = 1,\dots,m$) through the force $S_j = \sigma_j F_j$. We rewrite Eq. (5) in the form

$$S_j - (\pi/l_j)^2 E_j I_j + \tau_j S_j^\gamma (F_j E_j)^{1-\gamma} = 0. \tag{6}$$

Let the geometry of the structure be unambiguously described by a certain set of parameters. We combine them in the conditional vector $\bar{Z}$. The lengths of the rods and the forces in them are known (from the static problem) functions $\ell_j(\bar{Z})$, $S_j(\bar{Z})$. The remaining parameters in (6) are either unknown functions Z or constants (depending on the formulation of the problem). By changing Z within the space of permissible values, we can determine $\bar{Z}$ when a certain characteristic of the system is optimal.

For example, in the problem of minimizing the volume (weight) of a structure, it is necessary to find the extremum of the sum

$$V = \sum_{j=1}^{m} V_j, \tag{7}$$

where the volumes of the individual rods, expressed in terms of their lengths and cross-sectional areas, are determined from (6). In the general case, this equation is nonlinear and cannot be solved analytically. We will cite one special case. Let the moments of inertia I_j be independent of Z, i.e., be fixed for each rod. We exclude the unknown function $F_j(Z) = V_j/\ell_j$ from (6) and write

$$V_j = \left(\tau_j S_j^\gamma / (\pi^2 E I_j / l_j^2 - S_j)\right)^{1/(\gamma-1)} l_j/E.$$

We assume the critical time to be the same for all of the rods $\tau_j = \tau$. Here, τ is taken as a common factor from the sum (7). Thus, the condition of the minimum of V will be independent of its value and the coefficient φ.

*The critical time under creep conditions was obtained in [7] without allowance for [6] and coincides with well-known results.

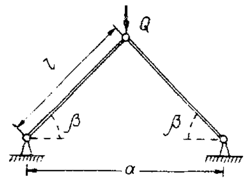

Fig. 1

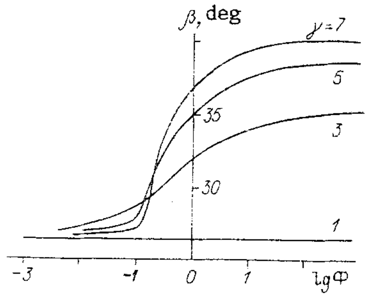

Fig. 2

Consequently, in the general case the optimum geometry is independent of the choice of instability criterion. It is easy to show that this is also generally valid for the problem of the critical time maximum (for all rods of the structure) with a fixed system volume. In fact, Eq. (7) will be the equation of the curve of $\tau(\bar{Z})$, where the individual volumes V_j are determined (numerically or analytically) from (6). The coefficient φ does not enter into this equation and thus affects only the value of t_{cr} (expressed from (4) in terms of φ) and not the optimum geometry $\bar{Z}$.

Let us present an example of the solution of a problem involving the optimization of a simple structure. The problem is formulated on the basis of the minimum weight of the structure. We will examine a system consisting of two rods (Fig. 1). The geometry vector here is unidimensional Z = β. We take the cross sections to be square: $F = b^2$, $I = b^4/12$. The dimension b is an unknown function of the parameter β. Excluding b from (6), we obtain

$$S - E(\pi V/l^2)^2/12 + \tau S^{\gamma}(l/(VE))^{\gamma-1} = 0. \tag{8}$$

We know the functions $S(\beta) = 0.5Q/\sin\beta$, $\ell(\beta) = 0.5a/\cos\beta$. We introduce the variable $x = \tan\beta$ and we rewrite (8) in the form

$$(xV)^{\gamma-1}(1 + x^2)^{1-\gamma} - KV^{\gamma+1}x^{\gamma}(1 + x^2)^{-\gamma-3/2} + M = 0. \tag{9}$$

Here $K = 8\pi^2E/(3a^4Q)$; $M = \tau(Qa/(4E))^{\gamma-1}$. The condition of the extremum V is the equality $dV/dx = 0$. We differentiate the last equation with allowance for this and obtain

$$(\gamma - 1)(1 - x^2) - KV^2x(1 + x^2)^{-5/2}(\gamma - x^2(\gamma + 3)) = 0. \tag{10}$$

Together with (9), Eq. (10) gives

$$V^{\gamma-1} = M(x + 1/x)^{\gamma-1}(\gamma - x^2(\gamma + 3))/(4x^2 - 1).$$

When V > 0, it follows from the above that x lies within a fairly narrow range of possible values $0.5 < x < \sqrt{\gamma/(\gamma + 3)}$. The size of the interval is determined only by the material parameter γ and is independent of both the acting loads and the chosen creep instability criterion. An exact value of x can be obtained from an equation which follows from (9) and (10)

$$\Phi(\gamma - x^2(\gamma + 3))^{(1+\gamma)/(\gamma-1)} = (\gamma - 1)(1 - x^2)(4x^2 - 1)^{2/(\gamma-1)}\, x(1 + x^2)^{-1/2} \tag{11}$$

($\Phi = KM^{2/(\gamma-1)}$). Figure 2 shows the solution (11) in relation to Φ for different γ. The horizontal asymptote of all of the curves is the straight line β = 26°34', which corresponds to the solution of the elastic problem and the case γ = 1.

We calculated β for a specific case to compare different instability criteria. Let α = 2/3, n = 5 (γ = 3), $\tau(\pi/a)^2Q/E = 800$. The corresponding values of β are shown in the last column of Table 1. It is evident that the effect of the choice of criterion on β is negligible.

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ASYMPTOTIC ANALYSIS OF PROBLEMS ON THE FREE VIBRATION OF RECTANGULAR TRANSVERSELY ISOTROPIC AND THREE-LAYER PLATES

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This article examines problems concerning the free vibration of transverse isotropic and three-layer rectangular plates (refined theory of bending accounting for shear through the thickness). The problems are described by a system of two equations, the first being of the order 2m (m = 2, 3 for transversely isotropic and three-layer plates, respectively) and the second a singularly perturbed second-order equation containing the small parameter ε. For transversely isotropic plates, ε characterizes the effect of transverse shears, while it characterizes the shear stiffness of the three-layer sandwich in the case of three-layer plates. We construct asymptotic expansions of the solutions with allowance for angular boundary-layer solutions, when the parameter ε is small. In this case, the second equation is a perturbation equation whose solution is in the nature of a boundary layer (edge effect).

Different types of boundary conditions are examined for the initial systems. We study the relationship between the boundary conditions of the initial and truncated problems (with the perturbation equation omitted). Substantiation is provided for the transition from the boundary conditions in the refined formulation to the classical formulation in the neighborhood of points of inflection (i.e., for a piecewise-smooth contour). Use of the Kirchhoff transform is validated for a free edge near a corner. Although a separation of variables is often possible for truncated problems, the complete system of equations does not permit such separation.

In the classical theory of the bending of plates, there is a contradiction between the overall order of the system of equations (two biharmonic equations for the normal deflection and the stream function) and five natural static boundary conditions. Thus, on the free edge, the bending and turning moments, the shearing force, and two forces in the plane of the plate are equal to zero. In the classical theory, four rather than five boundary conditions are established for the free edge if the Kirchhoff transform is used. There are theories which are refinements of the classical theory and make use of more general hypotheses in deriving the equations (allowance for shear through the plate thickness). The contradiction between the overall order of the system and the natural static boundary ocnditions disappears in these theories. The form of the system which is simplest for analytical purposes is probably that presented in [1, 2]. The order of this system is higher than in the classical theory due to the presence of a second-order equation having a solution of the edge-effect (boundary-layer) type.

Researchers have developed a method of changing over from the boundary conditions of the refined theory to the boundary conditions of the classical theory [3-5] (an example

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